SPINES OF FLEMING-VIOT PROCESSES

Krzysztof Burdzy
University of Washington
THEOREM (Grigorescu and Kang (2012); Bieniek and B (2018))

Every Fleming-Viot process has only one spine.
THEOREM (Grigorescu and Kang (2012); Bieniek and B (2018))

Every Fleming-Viot process has only one spine.
THEOREM (Bieniek and B (2018))

Suppose that the driving Markov process has a finite state space. Then, when the number of particles goes to infinity, the spine distribution converges to that of the driving Markov process conditioned to never hit the boundary.

THEOREM (B and Englander (2023))

The distribution spine of a Fleming-Viot process driven by Brownian motion in a bounded Lipschitz domain with the Lipschitz constant less than 1 converges to that of Brownian motion conditioned to never hit the boundary when the number of particles goes to infinity.
THEOREM (Bieniek and B (2018))

Suppose that the driving Markov process has a finite state space. Then, when the number of particles goes to infinity, the spine distribution converges to that of the driving Markov process conditioned to never hit the boundary.

THEOREM (B and Englander (2023))

The distribution spine of a Fleming-Viot process driven by Brownian motion in a bounded Lipschitz domain with the Lipschitz constant less than 1 converges to that of Brownian motion conditioned to never hit the boundary when the number of particles goes to infinity.
CONJECTURE

For every Fleming-Viot process, when the number of particles goes to infinity then the spine distribution converges to that of the driving process conditioned to never hit the boundary.
**THEOREM (B, Hołyst and March (2000))**

Consider a Fleming-Viot process driven by Brownian motion. For every fixed time \( t \), when the number of particles goes to infinity, their distribution converges to the distribution of Brownian motion conditioned to survive until \( t \).

---

**THEOREM (Villemonais (2014)); qualitative version**

For any Fleming-Viot process and any fixed time \( t \), when the number of particles goes to infinity, their distribution converges to the distribution of Brownian motion conditioned to survive until \( t \).
Consider a Fleming-Viot process driven by Brownian motion. For every fixed time $t$, when the number of particles goes to infinity, their distribution converges to the distribution of Brownian motion conditioned to survive until $t$.

For any Fleming-Viot process and any fixed time $t$, when the number of particles goes to infinity, their distribution converges to the distribution of the process conditioned to survive until $t$. 
From points to trajectories

**THEOREM/REMARK** (Bieniek and B (2018))

The Villemonais’ theorem applies to path-valued processes.

\[\text{(red, black, blue)} \rightarrow \text{(black, green, blue)} \rightarrow \text{(black, orange, green)}\]
THEOREM/REMARK (Bieniek and B (2018))

The Villemonais’ theorem:
“For any Fleming-Viot process and any fixed time $t$, when the number of particles goes to infinity, the distribution of uniformly chosen historical process converges to the distribution of the driving process conditioned to survive until $t$.”
Villemonais: $\left( \lim_{t \to \infty} \right) \lim_{n \to \infty}$

We need: $\lim_{n \to \infty} \lim_{t \to \infty}$
Villemonais: \((\lim_{t \to \infty}) \lim_{n \to \infty}\)

We need: \(\lim_{n \to \infty} \lim_{t \to \infty}\)

The spine is not chosen uniformly at time \(t\).
THEOREM (Villemonais (2014)); quantitative version

For any Fleming-Viot process and any fixed time $t$, when the number of particles goes to infinity, their distribution converges to the distribution of the process conditioned to survive until $t$. 
Path permutations
Remaining challenges

Branching.

Paths close to the boundary.
What is the distribution of the spine?
THEOREM (B and Tadić (2023))

The spine of the Fleming-Viot process with two particles in halfline is not distributed as the three-dimensional Bessel process.
Branching explosion?

$t_1 = 1$

$t_2 = \frac{3}{2}$

$t_3 = \frac{7}{4}$

$t_4 = \frac{15}{8}$
Branching explosion?

$t_2 = 3/2$

$t_1 = 1$

$t_3 = 7/4$

$t_4 = 15/8$

$t_k \rightarrow 2$  ???

Krzysztof Burdzy  
SPINES OF FLEMING-VIOT PROCESSES  
15 / 18
THEOREM (Bieniek, B and Pal (2012))

Fleming-Viot process on half-line driven by Brownian motion with a drift stronger than that of a Bessel process, with an arbitrary number of particles, will have a branching explosion.
THEOREM (Bieniek, B and Pal (2012))

Fleming-Viot process on half-line driven by Brownian motion with a drift stronger than that of a Bessel process, with an arbitrary number of particles, will have a branching explosion.

THEOREM (Kolb and Liesenfeld (2022))

For a certain value of the parameter, a two-particle Bessel process on half-line has a branching explosion but the three-particle version does not.
Fleming-Viot process driven by Brownian motion with an arbitrary number of particles will not have a branching explosion if
(i) the domain is Lipschitz domain with the Lipschitz constant less than 1, or
(ii) the domain is polyhedral.
THEOREM (Kwaśnicki (2020); unpublished)

Fleming-Viot process with two particles in a bounded domain driven by Brownian motion does not have a branching explosion.
Branching explosion? No

**THEOREM (Kwaśnicki (2020); unpublished)**

Fleming-Viot process with two particles in a bounded domain driven by Brownian motion does not have a branching explosion.

**CONJECTURE**

Fleming-Viot process with any number of particles in a bounded domain driven by Brownian motion does not have a branching explosion.